Fig. 1. Polar and cartesian parameterizations of the half-disc $D$.

**APPENDIX**

**DERIVATION OF THE DEFAULT $\gamma$ VALUE**

**Theorem 1:** Let $R$ be a positive real number. Let $w_p(x) = \left(1 - \min\left\{\frac{\|p - x\|}{R}, 1\right\}\right)^2$, for $p, x \in \mathbb{R}^3$, let $H$ be a half-plane and $v$ be a point on the boundary of $H$. Then

$$\frac{\|\int_{p \in H} w_p(v)(p - v)\|}{\sqrt{\int_{p \in H} w_p(v)\left(\int_{p \in H} w_p\|p - v\|^2\right) dH}} = \frac{512\sqrt{6}}{693\pi}.$$  \hspace{1cm} (1)

In the context of boundary detection, $v$ is a point to be classified, while $p$ is a splat with radius $R$.

**Proof:** Firstly note that if $\|p - v\| > R$ then $w_p(v) = 0$. Thus, if $D$ is the subset of $H$ within a distance $R$ of $v$, then we can replace $H$ by $D$ in the integrals in (1). We parameterize the half-disc $D$ using polar coordinates $(r, \theta)$, with $r \in [0, R]$ and $\theta \in [0, \pi]$, as shown in Fig. 1. We can also Cartesian coordinates $(x, y) = (r \cos \theta, r \sin \theta)$.

$$w(r) = \left(1 - \left(\frac{r}{R}\right)^2\right)^4 = 1 - \frac{4}{R^2} r^2 + \frac{6}{R^4} r^4 - \frac{4}{R^6} r^6 + \frac{1}{R^8} r^8.$$

Let us now consider the integrals one at a time. The integral in the numerator is a vector integral, but from symmetry it is clear that the $x$ component will vanish and we need only compute the $y$ component

$$\int_{H} w_p(v)(p - v)_y \, dH = \int_{r, \theta} w(r) r \sin \theta \cdot r \cdot dr \cdot d\theta = \left(\int_{0}^{R} r^2 w(r) \, dr\right) \left(\int_{0}^{\pi} \sin \theta \, d\theta\right) = R^3 \left(\frac{1}{3} - \frac{4}{5} + \frac{6}{7} - \frac{4}{9} + \frac{1}{11}\right) \cdot 2 = \frac{256}{3465} R^3.$$

Next, the integrals in the denominator:

$$\int_{H} w_p(v)\|p - v\|^2 \, dH = \int_{r, \theta} w(r) r^3 \cdot dr \cdot d\theta = \left(\frac{1}{3} - \frac{4}{5} + \frac{6}{7} - \frac{4}{9} + \frac{1}{11}\right) R^4 \pi = \frac{\pi}{60} R^4.$$

The left hand side of (1) is thus

$$\frac{256R^3}{\sqrt{\frac{\pi}{10} R^2 \cdot \frac{\pi}{60} R^4}} = \frac{256\sqrt{600}}{3465\pi} = \frac{512\sqrt{6}}{693\pi}.$$